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Image Restitution Using Non-Locally Centralized Sparse Representation Model

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Abstract
Sparse representation models uses a linear combination of a few atoms selected from an over-completed dictionary to code an image patch which have given good results in different image restitution applications. The reconstruction of the original image is not so accurate using traditional models of sparse representation to solve degradation problems which are blurring, noisy, and down-sampled. The goal of image restitution is to suppress the sparse coding noise and to improve the image quality by using the concept of sparse representation. To obtain a good sparse coding coefficients of the original image we exploit the image non-local self similarity and then by centralizing the sparse coding coefficients of the observation image to those estimates. This non-locally centralized sparse representation model outperforms standard sparse representation models in all aspects of image restitution problems including de-noising, de-blurring, and super-resolution.

I. INTRODUCTION

The main problem in image processing is image is degraded by the following versions down sampling, noisy, and blurring, such as medical imaging, remote sensing, close observation especially of a suspected spy(or)criminal, and entertainment, etc. For an observed image **y**, the problem of image restoration can be formulated by

$$y=Hx+\upsilon \quad (1)$$

Where **H** is a degradation matrix, x is the original image vector and υ is the additive noise vector. With different settings of matrix **H,** Eq. (1) can represent different image restitution problems; for example, image de-noising when **H** is an identity matrix, image de-blurring when **H** is a blurring operator, image super-resolution when **H** is a composite operator of blurring and down-sampling, and compressive sensing when **H** is a random projection matrix[1]-[3]. In the past decades, extensive studies have been conducted on developing various image restitution approaches[4]-[23],[28].Due to the loss of information caused by motion blur nature of image restitution, the regularization-based techniques have been widely used by regularizing the solution spaces[5]-[9],[12],[22].In order for an effective regularizer, it is of great importance to find and model the appropriate prior knowledge of natural images, and various image prior models have been developed[5]-[8], [14], [17], [18], [22].

Sparse representation is used to reconstruct original image from the degraded image. Sparse representation is a principle of that a image can be approximated by a linear combination of sparse codes.

It can be formulated as $b=x_1a_1+\ldots\ldots\ldots..+x_ka_k$

Where a_1,a_2 are dictionary atoms or basis vector

x_1, x_2.............are sparse co-efficient vector

The classic regularization models introducing additional information to solve the loss of information caused by the motion blur such as the quadratic Tikhonov regularization[8] and the TV regularization[5]-[7] are effective in removing the noise errors but have certain characteristics to over-smooth the images due to the piecewise constant assumption. As an uncommon, in recent years the sparsity–based regularization[9]-[23] had led to promising results for image restitution problems[1]-[3], [16]-[23].The sparse representation model assumes that image $X \in \mathbb{R}^N$ can be represented as $\mathbf{x} \approx \mathbf{\Phi\alpha}$, where $\mathbf{\Phi} \in \mathbf{R}^{n*M}$ **(N<M)** is an over-complete dictionary, and most entries of the coding vector α are zero or close to zero. The sparse decomposition of **x** can be obtained by solving an $\mathbf{l_o}$-minimization problem, formulated as $\alpha_x = \arg\min_{\alpha} \|\alpha\|_0, \text{s.t.} \|x-\Phi\alpha\|_2 \le \varepsilon$, where $\|\cdot\|_0$ is a false norm that counts the number of non-zero entries in $\boldsymbol{\alpha}$, and $\boldsymbol{\varepsilon}$ is a small constant controlling the approximation error. since $\mathbf{l_o}$-minimization is an NP-hard combinational optimization problem, it is often

relaxed to the convex **l_1**-minimization .The **l_1**-norm based sparse coding problem can be generally formulated in the following Lagrangian form:

$$\alpha_x = \arg\min_{\alpha} \left\{ \| x - \Phi\alpha \|_2^2 + \lambda \| \alpha \|_1 \right\} \qquad \textbf{(2)}$$

Where constant **λ** denotes the regularization parameter. With an appropriate selection or the regularization parameter **λ**, we can get a good balance between the sparse approximation error of **x** and the sparsity of **α** ,and the term "sparse coding" refer to this sparse approximation process of **x**. Many efficient **l_1**-minimization techniques have been proposed to solve Eq.(2), such as iterative thresholding algorithms [9]-[11] and breg-man split algorithms [24], [25].

In addition, compared with the analytically designed dictionaries (e.g. wavelet/curvelet dictionary),the dictionaries learned from example image patches can improve much the sparse representation performance since they outperforms characterize the image structures.[26], [27].

In the scenario of image restitution, what we observed is the degraded image signal y via. To recover **x** from **y**, first **y** is sparsely coded with respect to **Φ** by solving the following minimization problem:

$$\alpha_y = \arg\min_{\alpha} \| y - H\Phi\alpha \|_2^2 + \lambda \| \alpha \|_1 \qquad (3)$$

Here the concept of sparse coding noise is introduced. The difference between the sparse code of the degraded image and original sparse code image is sparse coding noise (SCN).

$$\boldsymbol{\upsilon_\alpha = \alpha_y - \alpha_x}$$

where $\boldsymbol{\upsilon_\alpha}$ sparse coding noise

$\boldsymbol{\alpha_y}$-is sparse code of degraded image

$\boldsymbol{\alpha_x}$-is sparse code of original image

The goal of image restoration turns to suppress the sparse coding noise. To reduce the sparse coding noise centralized the sparse codes to some good estimation of $\boldsymbol{\alpha_x}$

In practice, a good estimation of can be obtained by exploiting the rich amount of non-local redundancies in the observed image.

The proposed NCSR model can be solved effectively by traditional iterative shrinkage algorithm [9], which allows us to adaptively adjust the regularization parameters from a Bayesian viewpoint. The extensive experiments conducted on typical image restitution problems, including image de-noising, de-blurring and super-resolution, demonstrate that the proposed NCSR based image restitution method can achieve highly competitive performance to state-of-the-art de-noising methods(e.g.,BM3D[17], [39]-[41], LSSC[18]), and outperforms state-of –the-art image de-blurring and super-resolution methods.

II. NON-LOCALLY CENTRALIZED SPARSE REPRESENTATION (NCSR)

Following the notation used in [19], for an image **x ϵ $\mathbf{R}^N$** let **x_i=R_ix** denote an image patch of size $\sqrt{n} * \sqrt{n}$ extracted at location i, where **R_i** is the matrix extracting patch **x_i** from x at location i. Given an dictionary **Φ ϵ $\mathbf{R}^{n*M}$**, n ≤ M each patch can be sparsely represented as **x_i≈ Φ $\alpha_{x,i}$** by solving an **l_1**-minimization problem **$\alpha_{x,i}$ = $\text{argmin}_{\alpha i}$\{‖x_i-Φα_i‖$_2^2$+λ‖α_i‖1 \}**. Then the entire image **x** can be overlapped to suppress the boundary errors, and we obtain a redundant patch-based representation.

Reconstructing x from **\{$\alpha_{x,i}$\}** is an over-determined system, and a straightforward least-square solution is [19]:.

$$x \approx \left(\sum_{i=1}^{N} R_i^T R_i \right)^{-1} \sum_{i=1}^{N} \left(R_i^T \Phi \alpha_{x,i} \right)$$

For the convenience of expression, we let

$$x \approx \Phi \circ \alpha_x = \left(\sum_{i=1}^{N} R_i^T R_i \right)^{-1} \sum_{i=1}^{N} \left(R_i^T \Phi \alpha_{x,i} \right) \qquad (4)$$

where $\alpha_x$denotes the concatenation of all $\alpha_{x,i}$. The above equation is nothing but telling that the overall image is reconstructed by averaging each reconstructed patch of x_i.

In the scenario of image restitution (IR), the observed image is modeled as y=Hx+. The sparsity-based image restitution method recovers **x** from **y** by solving the following minimization problem.

$$\alpha_y = \arg\min_{\alpha} \| y - H\Phi\alpha \|_2^2 + \lambda \| \alpha \|_1 \qquad (5)$$

The image **x** is then reconstructed as $\hat{x} = \Phi \circ \alpha_y$

A. Sparse coding noise

In order for an effective image restitution , the sparse codes obtained by solving the objective function in Eq.(5) are expected to be as close as possible to the true sparse codes of the original image x. However, due to the degradation of the observed image y(e.g., noisy and blurred), the image restitution quality depends on the level of the sparse coding (SCN), which is defined as the difference between $\boldsymbol{\alpha_y}$ and $\boldsymbol{\alpha_x}$

$$\upsilon_\alpha = \alpha_y - \alpha_x \qquad (6)$$

In the first experiment, we add Gaussian white noise to original image **x** to get the noisy image **y** (the noise level σ_n =15).Then we compute $\boldsymbol{\alpha_x}$ and $\boldsymbol{\alpha_y}$ by solving Eq.(2)and Eq(5), respectively. The Discrete Cosine Transform bases are adopted in the experiment. Then the sparse coding noise $\boldsymbol{\upsilon_\alpha}$ is computed. we plot the distribution of $\boldsymbol{\upsilon_\alpha}$ corresponding to the 4th atom in the dictionary. we plot the distributions of when the observed data y is blurred(by a Gaussian blur kernel with standard deviation 1.6) and down-sampled by factor 3 in both horizontal and vertical directions (after blurred by a Gaussian blur kernel with standard deviation 1.6), respectively. We can see that the empirical distributions of sparse coding noise $\boldsymbol{\upsilon_\alpha}$ can be well characterized by Laplacian distributions, while the Gaussian distributions have much larger fitting errors.

B. Modeling of NCSR

The definition of sparse coding noise indicates that by suppressing the sparse coding noise $\boldsymbol{\upsilon_\alpha}$ we could improve the image restitution output $\hat{x}$. However, the difficulty lies in that the sparse coding vector $\boldsymbol{\alpha_x}$ is unknown so that $\boldsymbol{\upsilon_\alpha}$ cannot be directly measured. Nonetheless, if we could have some reasonably good estimation of $\boldsymbol{\alpha_x}$, denoted by $\boldsymbol{\beta}$ available, then $\boldsymbol{\alpha_y} - \boldsymbol{\beta}$ can be a good estimation of the sparse coding noise $\boldsymbol{\upsilon_\alpha}$. To suppress $\boldsymbol{\upsilon_\alpha}$ and improve the accuracy of $\boldsymbol{\alpha_y}$ and further improve the objective function of Eq.(5),we can propose the following centralized sparse representation model[22]:

$$\alpha_y = \arg\min_\alpha \left\{ \| y - H\Phi \circ \alpha \|_2^2 + \lambda \sum_i \| \alpha_i \|_1 + \gamma \sum_i \| \alpha_i - \beta_i \|_p \right\} \quad (7)$$

Where $\boldsymbol{\beta_i}$ is some good estimation of $\boldsymbol{\alpha_i}$, $\boldsymbol{\gamma}$ is the regularization parameter and p can be 1 or 2.In the above centralized sparse representation model, while enforcing the sparsity of coding coefficients the sparse codes are also centralized to some estimate of so that sparse coding noise can be suppressed.

One important issue of sparsity-based image restitution is the selection of dictionary .conventional analytically designed dictionaries, such as discrete cosine transform, wavelet and curvelet dictionaries, are in sufficient to characterize the so many complex structures of natural images. The universal dictionaries learned from example image patches by using algorithms such as KSVD[26] can better adapt to local image structures. In general the learned dictionaries are required to be very redundant such that they can represent various image local structures. However, it has been shown that sparse coding with an over-complete dictionary is unstable[42], especially in the scenario of image restitution. In our previous work[21], we cluster the training patches extracted from a set of example images into K clusters, and learn a PCA sub-dictionary is adaptively selected to code it, leading to a more stable and sparse representation, and consequently better image restitution results.

We extract image patches from image x and cluster the patches into K clusters (K=70) by using the K- means clustering method. Since the patches in a cluster are similar to each other, there is no need to learn an over-complete dictionary for each cluster. Therefore, each cluster we learn a dictionary of PCA bases and use this compact PCA dictionary to code the patches in this cluster.(For the details of PCA sub-dictionaries construct a large over-complete dictionary to characterize all the possible local structures of natural images.

In the conventional sparse representation models as well as the model in Eq.(7),the local sparsity term $\|\boldsymbol{\alpha_i}\|_1$ is used to ensure that only a small number of atoms are selected from the over-complete dictionary Φ to represent the input image patch. In our algorithm for each patch to be coded, we adaptively select one sub-dictionary from the trained K PCA sub-dictionaries to code it . This is actually enforces the coding coefficients of this patch over the other sub-dictionaries to be 0, leading to a very sparse representation of the given patch. In other words, our algorithm will naturally ensure the sparsity of the coding coefficients, and thus the local sparsity of the coding coefficients, and thus the local sparsity regularization term $\|\boldsymbol{\alpha_i}\|_1$ can be removed. Hence we propose the following sparse coding model:

$$\alpha_y = \arg\min_\alpha \left\{ \| y - H\Phi \circ \alpha \|_2^2 + \lambda \sum_i \| \alpha_i - \beta_i \|_p \right\} \quad (8)$$

There is only one regularization term $\|\boldsymbol{\alpha_i} - \boldsymbol{\beta_i}\|_p$ in the above model. In the above model. In the case that p=1, and the estimate $\boldsymbol{\beta_i}$ is obtained by using the non-local redundancy of natural images, this regularization term will become a non-locally centralized sparse representation(NCSR).Next lets discuss how to obtain a good estimation $\boldsymbol{\beta_i}$ of the unknown sparse coding vectors $\boldsymbol{\alpha_i}$.

C. Non-local Estimate of Unknown Sparse code

Generally, there can be various ways to make an estimate of $\boldsymbol{\alpha_x}$, depending on how much the prior knowledge of $\boldsymbol{\alpha_x}$ we have. If we have many training images that are similar to the original image **x**, we could learn the estimate $\boldsymbol{\beta}$ of $\boldsymbol{\alpha_x}$ of from the training set. However, in many practical situations the training images are simply not available. On the other hand, the strong non-local correlation between the sparse coding coefficients allows us to learn the estimate $\boldsymbol{\beta}$ from the input data. Based on the fact that natural images often contain repetitive structures, i.e., the rich amount of non-local redundancies [30], we search the non-local similar patches to the given patch i in a large window centered at pixel i. For higher performance, the search of similar patches can also be carried out across different scales at the expense of higher computational complexity, as shown in [31]. Then a good estimation n of $\boldsymbol{\alpha}_i$, i.e., $\boldsymbol{\beta}_i$, can be computed as the weighted average of those sparse codes associated with the non-local similar patches (including patch i) to patch i. For each patch $\mathbf{x}_i$, we have a set of its similar patches, denoted by Ω_i. Finally $\boldsymbol{\beta}_i$ can be computed from the sparse codes of the patches within Ω_i.

Denote by $\boldsymbol{\alpha}_{i,q}$ the sparse codes of patch $\mathbf{x}_{i,q}$ within set Ω_i then can be computed as the weighted average of $\boldsymbol{\alpha}_{i,q}$

$$\beta_i = \sum_{q\in\Omega_i} \omega_{i,q}\alpha_{i,q} \quad (9)$$

Where $w_{i,q}$ is the weight. Similar to the non-local means approach[30], we set the weights to be inversely proportional to the distance between patches $\mathbf{x}_i$ and $\mathbf{x}_{i,q}$

$$w_{i,q} = \frac{1}{W}\exp\left(-\|\hat{x}_i - \hat{x}_{i,q}\|_2^2 / h\right) \quad (10)$$

Where $\hat{x}_i = \Phi\hat{\alpha}_i$ and $\hat{x}_{i,q} = \Phi\hat{\alpha}_{i,q}$ are the estimates of the patches x_i and $x_{i,q}$, h is a pre-determined scalar and W is the normalization factor. In the case of orthogonal dictionaries(e.g., the local PCA dictionaries used in this work), the sparse codes $\hat{\alpha}_i$ and $\hat{\alpha}_{i,q}$ can be easily computed as $\hat{\alpha}_i = \Phi^T\hat{x}_i$ and $\hat{\alpha}_{i,q} = \Phi^T\hat{x}_{i,q}$. Our experimental results show that by exploiting the non-local redundancies of natural images, we are able to achieve good estimation of the unknown sparse vectors and the NCSR model of Eq.(8) can significantly improve the performance of the sparsity-based image restitution results.

Eq.(8) can be solved iteratively. We first initialize $\boldsymbol{\beta}_i$ as 0, i.e., $\beta_i^{(-1)} = 0$ and solve for the sparse coding vector, denoted by $\alpha_y^{(0)}$, using some standard sparse coding algorithm. Then we can get the initial estimation of **x**, denoted by $x^{(0)}$, via $x^{(0)} = \Phi\circ\alpha_y^{(0)}$. Based on $x^{(0)}$, we search for the similar patches to each patch i, and hence the non-local estimate of $\boldsymbol{\beta}_i$ can be updated using Eqs.(9) and (10). The updated estimation of $\boldsymbol{\alpha}_x$ denoted by $\beta_i^{(0)}$, will then be used to improve the image restitution quality. Such a procedure is iterated until convergence. In the $\mathbf{l}^{th}$ iteration, the sparse vector is obtained by solving the following minimization problem .

$$\alpha_y^{(l)} = \arg\min_{\alpha} \left\{\|y - H\Phi\circ\alpha\|_2^2 + \lambda\sum_i \|\alpha_i - \beta_i^l\|_p\right\} \quad (11)$$

The restorecd image is then updated as. $\hat{x}^{(l)} = \Phi\circ\alpha_y^{(l)}$.

In the above iterative process, the accuracy of sparse coding coefficient $\alpha_y^{(l)}$ is gradually improved, which in turn improves the accuracy of $\boldsymbol{\beta}_i$. The improved $\boldsymbol{\beta}_i$ are then used to improve the accuracy of $\boldsymbol{\alpha}_y$ and so on. Finally, the desired sparse code vector is obtained when the alternative optimization process falls into a local minimum.

III. ALGORITHM OF NCSR

A. parameters determination

In Eq.(8) or Eq.(11) the parameter **λ** that balances the fidelity term and the centralized sparsity term should be adaptively determined for better image restitution performance. In this subsection we provide a Bayesian interpretation of the image restitution using non-locally centralized sparse representation model, which also provides us an explicit way to set regularization parameter **λ**. In the literature of wavelet de-noising, the connection between Maximum a Posterior(MAP) estimator and sparse representation has been established [28], and here we extend the connection from the local sparsity to non-locally centralized sparsity.

For the convenience of expression, let's define **θ = α -β**

For a given **β**, the MAP estimation of **θ** can be formulated as

$$\theta_y = \arg\max_{\theta} \log P(\theta / \mathrm{y}) = \arg\max_{\theta}\{\log P(\mathrm{y}/\theta) + \log \mathrm{P}(\theta)\} \quad (12)$$

The likelihood term is characterized by the Gaussian distribution

$$P(y/\theta) = P(y/\alpha,\beta) = \frac{1}{\sqrt{2\pi}\sigma_n}\exp\left(-\frac{1}{2\sigma_n^2}\|y - H\Phi\circ\alpha\|_2^2\right) \quad (13)$$

Where **θ** and **β** are assumed to be independent. In the prior probability **P(θ)** , **θ** reflects the variation of from its estimation **β** . If we take **β** as a very good estimation of the sparse coding coefficient of unknown true signal, then $\boldsymbol{\theta}_y = \boldsymbol{\alpha}_x - \boldsymbol{\beta}$ is basically the sparse coding noise associated with $\boldsymbol{\alpha_y}$, and the sparse coding noise signal can be well characterized by the Laplacian distribution. Thus, we can assume that θ follows i.i.d. Laplacian distribution, and the joint prior distribution **P(θ)** can be modeled as

$$P(\theta) = \prod_i \prod_j \left\{ \frac{1}{\sqrt{2}\sigma_{i,j}} \exp\left(-\frac{|\theta_i(j)|}{\sigma_{i,j}} \right) \right\} \quad (14)$$

where $\theta_i(j)$are the j[th] elements of θ_i, and $\sigma_{i,j}$ is the standard deviation of $\theta_i(j)$substituting Eqs. (13) and (14) into Eq. (12), we obtain

$$\theta_y = \arg\min_\theta \left\{ \| y - H\Phi\circ\alpha \|_2^2 + 2\sqrt{2}\sigma_n^2 * \sum_i \sum_j \frac{1}{\sigma i, j} |\alpha_i(j) - \beta_i(j)| \right\} \quad (15)$$

Hence, for a given **β** the sparse codes **α** can then be obtained by minimizing the following objective function

$$\alpha_y = \arg\min_\alpha \left\{ \| y - H\Phi\circ\alpha \|_2^2 + 2\sqrt{2}\sigma_n^2 * \sum_i \sum_j \frac{1}{\sigma_{i,j}} |\alpha_i(j) - \beta_i(j)| \right\} \quad (16)$$

Compared with Eq. (8) we can see that the $\mathbf{l_1}$-normalization(i.e., p=1) should be chosen to characterize the sparse coding noise term **αi-βi** comparing Eq. (16) with (8),we have

$$\lambda_{i,j} = \frac{2\sqrt{2}\sigma_n^2}{\sigma_{i,j}} \quad (17)$$

In order to have robust estimations of $\sigma_{i,j}$ the image non-local redundancies can be exploited. In practice, we estimate $\sigma_{i,j}$ using the set of $\boldsymbol{\theta_i}$ computed from the non-local similar patches $\boldsymbol{\lambda_{i,j}}$ with the updated with the updated **θ** in each iteration or in several iterations to save computational cost. Next we present the detailed algorithm of the proposed image restitution using non-locally centralized sparse representation scheme.

Algorithm : Image Restitution Using Non-locally Centralized Sparse Representation

1. Initialization:

(a) Set the initial estimate as $\hat{x}$ = y for image de-nosing and de-blurring, or initializing $\hat{x}$ by bi-cubic interpolator for image super-resolution;

(b) Set initial regularization parameter λ and δ;

2. Outer loop (dictionary learning and clustering): iterate on l=1, 2,....., L

(a) Update the dictionaries {Φ_k} via k-means and principle component analysis;

(b) Inner loop (clustering): iterate on j = 1,2,....., j

(I) $\hat{x}^{(j+1/2)} = \hat{x}^{(j)} + \delta H^T \left(y - H\hat{x}^{(j)} \right)$

where δ is the pre-determined constant;

(II)**Compute**

$$v^{(j)} = [\Phi_{k1}^T R_1 \hat{x}^{(j+1/2)}, \ldots\ldots, \Phi_{kN}^T R_N \hat{x}^{(j+1/2)}]$$

Where Φ_{k_i} **is the dictionary assigned to patch** $\hat{x}_i = R_i \hat{x}^{(j+1/2)}$;

(III)**Compute** $\alpha_i^{(j+1)}$ **using the shrinkage operator given in Eq.(19);**

(IV)**If mod (j, J_0) = 0 update the parameters $\lambda_{i,j}$ and {β_i}using Eqs. (17) and (9), respectively ;**

(V) **Image estimate update:** $\hat{x}^{(j+1)} = \Phi \circ \alpha_y^{(j+1)}$ **using Eq. (4)**

B. Iterative Shrinkage Algorithm

we use an iterative algorithm to solve the NCSR objective function in Eqs. (8) or (16). In each iteration, for fixed $\boldsymbol{\beta_i}$ we solve the following l_1-norm minimization problem

$$\alpha_y = \arg\min_\alpha \left\{ \| y - H\Phi\circ\alpha \|_2^2 + \sum_i \sum_j \lambda_{i,j} |\alpha_i(j) - \beta_j(\mathrm{j})| \right\}$$

(18)

Which is convex and we can be solved efficiently. In this paper we adopt the surrogate algorithm in [9] to solve Eq.(18). In the (l+1)-th iteration, the proposed shrinkage operator for the j[th] element of α_i is

$$\alpha_i^{(l+1)}(j) = S_\tau(\mathrm{v}_{i,j}^l - \beta_i(\mathrm{j})) + \beta_i(j)$$

(19)

Where $S_\tau(\cdot)$ is the classic soft-thresholding operator and $v^{(l)} = K^T(\mathrm{y} - \mathrm{K}\circ\alpha^{(l)}) / \mathrm{c} + \alpha^{(l)}$,where $K = H\Phi$, $K^T = \Phi^T \circ H^T$,

$\tau = \lambda_{i,j/c}$ and c is an auxiliary parameter guaranteeing the convexity of the surrogate function. The derivation of the above shrinkage operator follows the standard surrogate algorithm in [9]. The interesting readers may refer to [9]. The interesting readers may refer to [9] for details

C. Summary of the Algorithm

In our NCSR algorithm the adaptive sparse domain strategy [21] is used to code each patch. We cluster the patches of image x into K clusters and learn a PCA sub-dictionary Φ_k for each cluster. For a given patch, we first check which cluster it falls into by calculating its distances to means of the clusters, and then select the PCA sub-dictionary of the cluster to code it. The proposed NCSR based image restitution algorithm is summarized in Algorithm

For fixed parameters $\lambda_{i.j}$ and $\{\beta_i\}$ the the objective function in Eq.(18) is convex and can be be efficiently solved by the iterative shrinkage algorithm in the inner loop, and its convergence has been well established in[9]. Since we update the regularization parameter $\lambda_{i.j}$ and $\{\beta_i\}$ in every J0 iterations after solving a sub-optimization problem, algorithm is empirically convergent in general, as those presented in[38]

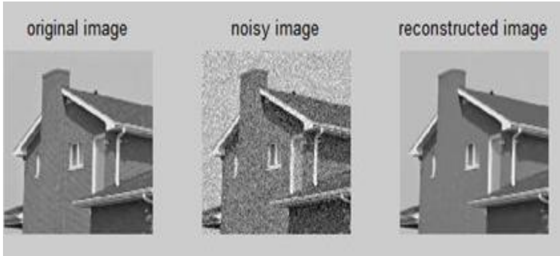
IV. EXPERIMENTAL RESULTS

To verify the image restitution performance of the proposed NCSR algorithm we conduct extensive experiments on image de-noising, de-blurring and super-resolution. The basic parameter setting of NCSR is as follows: the patch size7*7 is and K=70. For image de-noising, δ=0.02, L=3, and J=3; for image de-blurring and super-resolution, δ=2.4, L=5, and J=160. To evaluate the quality of the restitution images, the PSNR and the recently proposed powerful perceptual quality metric FSIM [32] are calculated.

A. Image De-noising

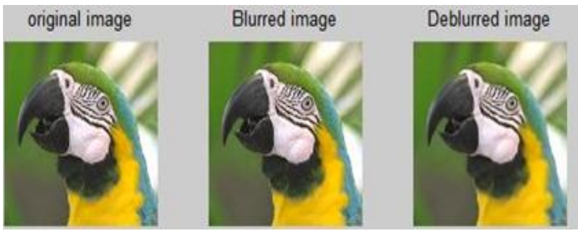
A set of 12 natural images commonly used in the literature of image de-noising are used for the comparison study. We can see that the proposed NCSR achieves highly competitive de-noising performance. We show the de-noising results on two typical images with moderate noise corruption and strong noise corruption, respectively. It can be seen that NCSR is very effective in reconstructing both the smooth and the texture/edge regions.

All the four competing methods can achieve very good de-noising outputs. In particular, the de-noising image by the proposed NCSR has much less errors than other methods, and is visually more pleasant.



B. Image De-blurring

We applied de-blurring methods to both the simulated blurred images and real motion blurred images. In the simulate image de-blurring two commonly used blur kernels, i.e., 9*9 uniform blur and 2D Gaussian function(non-truncated) with standard deviation 1.6, are used for simulations. Additive Gaussian noise with noise levels $\sigma_n = \sqrt{2}$ is added to the blurred images. In addition, 6 typical non-blind de-blurring image experiments presented in [36] and [41] are conducted for further test. For the real motion blurred images, we borrowed the motion blur kernel estimation method from [34] to estimate the blur kernel and then fed the estimated blur kernel into the NCSR de-blurring method. For color images, we only apply the de-blurring operation to the luminance component. We also test the proposed NCSR de-blurring method on real motion blurred images. Since the blur kernel estimation is a non-trivial task, we borrowed the kernel estimation method from [34] to estimate the blur kernel and apply the estimated blur kernel in NCSR to restitution the original images. We can see that the images restitution by our approach are much clearer and much more details are recovered. Considering that the estimated kernel will have bias from the true unknown blurring kernel, these experiments validate that NCSR is robust to the kernel estimation errors.



C. Image Super-resolution

In image super-resolution the simulated LR image is generated by first blurring an HR image with a 7*7 Gaussian kernel with standard deviation 1.6, and then down-sampling the blurred image by a scaling factor 3 in both horizontal and vertical directions. The additive Gaussian noise of standard deviation 5 is also added to the LR images, making the image restitution problem more challenging. Since human visual system is more sensitive to luminance changes, we only apply the image restitution methods to the luminance component and use the simple bicubic interpolator for the chromatic components. The NCSR approach reconstruct the best visually pleasant HR images.

V. CONCLUSION

In this paper we presented a novel image restitution using non-locally centralized sparse representation model. The sparse coding noise(SCN), which is defined as the difference between the sparse code of the unknown original image, should be minimized to improve the performance of sparsity-based image restitution. To this end, we proposed a centralized sparse constraint, which exploits the image non-local redundancy, to reduce the SCN. The Bayesian interpretation of the NCSR model was provided and this endows the NCSR model iteratively reweighted implementation. An efficient iterative shrinkage function was presented for solving the l1-regularized NCSR model an iteratively reweighted implementation. An efficient iterative shrinkage function was presented for solving the l1-regularized NCSR minimization problem. Experimental results on image de-noising, de-blurring and super-resolution demonstrated that the NCSR approach can achieve highly competitive performance to other leading de-noising methods, and outperform much other leading image de-blurring and super-resolution methods.

REFERENCES

[1] W. Dong, L. Zhang and G. Shi, "*Centralized sparse representation for image restoration* ," in *Proc. IEEE Int. Conf. on Computer Vision (ICCV)*, 2011.

[2] W. Dong, L. Zhang, G. Shi, and X. Wu, "*Image de- blurring and super-resolution by adaptive sparse domain selection and adaptive regularization*," *IEEE Trans. On Image Processing*, vol. 20, no. 7, pp.1838-1857, July 2011

[3] Jianchao Yang, John Wright, Thomas Huang and Yi Ma, "*Image Super-Resolution via Sparse Representation*", *IEEE Trans. Image Process.*, vol. 19, no. 11, pp. 2861-2873, Nov. 2010.

[4] A. Marquina, and S. J. Osher, "*Image super-resolution by TV-regularization and Bregman iteration*," *J.Sci. Comput.*, vol. 37, pp. 367-382, 2008.

[5] E. Candès and T. Tao, "*Near optimal signal recovery from random projections: Universal encoding strategies?*" *IEEE Trans. on Information Theory*, vol. 52, no. 12, pp. 5406 - 5425, December 2006.

[6] D. Donoho, "*Compressed sensing*," *IEEE Trans. on Information Theory*, vol. 52, no. 4, pp. 1289-1306,April 2006.

[7] E. Candès, J. Romberg, and T. Tao, "*Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," IEEE Trans. on Information Theory*, vol. 52, no. 2, pp. 489 - 509, February 2006.

[8] M Bertero and P. Boccacci, *Introduction to Inverse Problems in Imaging*. Bristol, U.K.: IOP, 1998.

[9] L. Rudin, S. Osher, and E. Fatemi, "*Nonlinear total variation based noise removal algorithms," Phys. D,*vol. 60, pp. 259–268, 1992.

[10] T. Chan, S. Esedoglu, F. Park, and A. Yip, "*Recent developments in total variation image restoration,"* in *Mathematical Models of Computer Vision*, N. Paragios, Y. Chen, and O.Faugeras, Eds. New York:Springer Verlag, 2005

[11] J. Oliveira, J. M. Bioucas-Dias, and M. Figueiredo, "*Adaptive total variation image de-blurring: a majorization-minimization approach*," *Signal Processing*, vol. 89, no. 9, pp. 1683-1693, Sep. 2009.

[12] A. N. Tikhonov, "*Solution of incorrectly formulated problems and regularization method*," *In Soviet Math. Dokl.*, vol. 4, pp. 1035-1038, 1963.

[13] I. Daubechies, M. Defriese, and C. DeMol, "*An iterative thresholding algorithm for linear inverse problems with a sparsity constraint*," *Commun. Pure Appl. Math*., vol.57, pp.1413-1457, 2004.

[14] P. Combettes, and V. Wajs, "*Signal recovery by proximal forward-backward splitting*," *SIAM J.Multiscale Model.Simul.*, vol.4, pp.1168-1200, 2005

[15] M. Zibulevsky and M. Elad, "*l1-l2 optimization in signal and image processing*," *IEEE SignalProcessing Magazine*, vol. 27, no. 3, pp. 76-88, May, 2010

[16] J. A. Tropp and S. J. Wright, "*Computational methods for sparse solution of linear inverse*